PROBLEM SET SEVEN: (PRE)SEMILATTICES AND TREES

Problem 1

Prove the three facts about $\sqcap$ when $\sqsubseteq$ is an order, and the first half of Interdefinability (slides on Review of Preorders, slide 34).

Problem 2

Prove that in a pregroupoid, if $a \equiv b$ and $c \equiv d$, then $a \circ c \equiv b \circ d$.

Problem 3

a. Prove that in an upper presemilattice, $\circ$ is a lub operation.

b. Prove that if $\circ$ is a lub operation on a preorder $\langle P, \sqsubseteq \rangle$, then $\langle P, \sqsubseteq, \circ \rangle$ is an upper presemilattice.

Problem 4

Fill in the missing details in the proof of Theorem 1.

Problem 5

Give the rest of the proof of Theorem 2.

Problem 6

Prove Theorem 4.

Problem 7

Prove the Corollary to Theorem 5. (You can take Theorem 5 to have been established.)

Problem 8

Prove Theorem 6.

Problem 9

Prove Theorem 7

Problem 10

Prove Theorem 8.