The Dynamics of Sense and Implicature: Anaphora, "Presupposition," and CIs

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A Bit about Me

- I finished up my Ph.D. thesis at Ohio State this past summer, advised by Carl Pollard and Craige Roberts
- I'm interested in logical/mathematical approaches to language, especially semantics and pragmatics, and applications to natural language processing (paraphrase alignment, generation)
- Currently I'm a research scientist at Nuance's recently established natural language understanding laboratory in Sunnyvale
- I also have a background in languages, philosophy, and software engineering

What's Happening at Nuance?

We're trying to go from this ...



What's Happening at Nuance?

To this:



What's Really Happening at Nuance?

- We're building conversational user agents that are intelligent
- This of course means using big data and machine learning techniques
- But we're also trying to leverage (what I'll call) *smart data*: morphology, syntax, semantics, discourse analysis, parsing, reasoning, AI techniques, ...

My Thesis in a Nutshell

- My dissertation work is about a new approach to semantics and pragmatics based on a novel way of characterizing meaning that has some old roots
- This new characterization leads to a generalized account of contextual felicity

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- My dissertation work is about a new approach to semantics and pragmatics based on a novel way of characterizing meaning that has some old roots
- This new characterization leads to a generalized account of contextual felicity
- I also give an explicit formal account of some English data based on this new taxonomy, encoded in a discourse semantics that captures both anaphora and Potts's (2005) "CIs"
- Its central feature is that foreground and background meaning are computed in parallel, and it is designed with computational applications in mind

Outline

Characterizing Sense and Implicature What's the Difference? A Gricean Taxonomy Felicity, Accommodation, and Variability A More Fully Fleshed-out Picture

Formalizing these Ideas

Technical Background Going Dynamic

Accounting for some Implicature Data

Anaphora Supplements

Conclusions

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Sense and Implicature

 Partly following Simons et al. (2010), the meanings of natural language utterances can be separated into

> Sense Literal meaning; what is asserted or proffered; the "main point"

Implicature Background implication; not central to main point; sometimes not even stated

Sense and Implicature

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- Senses are targeted by the semantic *operators* negation (*not*), modals (*might, maybe*), question words (*who/what/when/where/how, Did ...*), conditionals (*If ..., then ...*)
- But implicatures are not targeted by these operators: they are persistent

Spotting Implicatures I

One test for implicatures is embedding in the scope of an operator.

(1) Maybe the woman bought a ticket. (anaphora)
a. ≁ The woman bought a ticket.
b. ~ *The woman* has a retrievable antecedent.
(similarly for other definites: pronouns, proper names, etc.)

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 It's not true that Lance, who's a cyclist, is from Texas. (supplement)
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(similarly for parentheticals, non-restrictive relatives)

(3) Did Kim quit smoking?

(aspectual)

a. $\not\rightsquigarrow$ Kim used to smoke.

(similarly for *continue*, *start*, *stop*, *switch to*; achievements (*graduate*, *win*); "factives" (*know*, *realize*, *regret*))

Spotting Implicatures II

Another test for implicatures is direct acceptance/denial.

- (4) The woman bought a ticket.
 - a. No she didn't. $\not\sim$ The woman bought a ticket.
 - b. Yes/No. ~> *The woman* has a retrievable antecedent.

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- (5) Is Lance, who's a cyclist, from Texas?
 a. No. ≁ Lance is from Texas.
 b. Yes/No. ~ Lance is a cyclist.
- (6) Kim quit smoking.
 - a. Yes, that's true. \rightsquigarrow Kim used to smoke.
 - b. No. \rightsquigarrow Kim did not quit smoking.
 - c. No, she's never smoked in her life. $\not\leadsto$ Kim used to smoke.

"Gricean Implicature" I

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"Gricean Implicature" II

But what about the others? Grice didn't say too much about those ...



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I'll argue that implicatures can be characterized based on two criteria:

- 1. Whether their persistence is conventionally signaled, and
- 2. Whether they must be *speaker commitments*, a cross-cutting distinction

Conventional Implicature: Anaphora and "CIs"

The retrievability implication associated with anaphora has to be a speaker commitment:

(7) Kim doesn't know that there's a donkey_i over there. She doesn't hear it_i braying.

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But the non-anaphoric conventional implicatures do not:

(8) I'm a big Obama supporter. But my tea party neighbor thinks that Obama, who's totally a Kenyan pinko in charge of secret terrorist camp in the ravine behind his house, will destroy the country. (cf. Amaral et al., 2007; Harris and Potts, 2009)

Nonconventional Implicatures I: Achievements

Nonconventional implicatures can sometimes persist. A case in point is the preparatory phase associated with *achievements*.

- In (9), the speaker is committed to Lance's having participated in the Tour:
 - (9) Lance won the Tour de France.

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- In (9), the speaker is committed to Lance's having participated in the Tour:
 - (9) Lance won the Tour de France.
- But in (10), it can't be the speaker's commitment:
 - (10) Lance didn't enter last year's Tour de France, but Kim is convinced that he won it.

And the implication that Lance participated does not persist.

Nonconventional Implicatures II: Aspectuals

A second case is the *aspectuals*.

- As for the achievements, (11) entails that Kim used to drink caffeinated coffee:
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 - (12) I wonder why Kim is so sluggish lately. Maybe she switched to decaf, or something.

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- But again, depending on context, the entailment may not persist:
 - (12) I wonder why Kim is so sluggish lately. Maybe she switched to decaf, or something.
- And just as for the achievements, aspectuals do not have to be speaker commitments:
 - (13) Kim never drank caffeinated coffee, but Robin believes that Kim switched to decaf.

Nonconventional Implicatures III: "Factives"

So-called "factives" also exhibit similar behavior.

- Several authors have commented that factives aren't presuppositional when embedded beneath certain operators:
 - (14) Perhaps she just discovered that he's having an affair. (Simons, 2001)

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- But even in unembedded contexts, factives don't presuppose their complements:
 - (15) (*Driver to hitchhiker s/he just picked up*) Do you realize there's a gum wrapper in your hair?
- They can also be non-speaker commitments:
 - (16) The Riemann hypothesis remains a mysterious, unsolved conjecture in mathematics, but Louie just knows it is true.

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- In the case of supplements, they are infelicitous when their content conflicts with prior discourse (Potts's *nondeniability*):
 - (18) Lance, a cyclist, is from Texas. # Lance is not a cyclist.
- And similarly for the nonconventional implicatures:
 - (19) # Kim never smoked in her life, and then she stopped smoking.

What About Accommodating those "Presuppositions"?

- Members of the class of nonconventional implicatures (achievements, aspectuals, factives) are characterized as potentially, but not necessarily, giving rise to entailments
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- This taxonomy is at odds with theories that construe achievements, aspectuals, and factives as presuppositions that require accommodation when they contain new information
- In my proposal, accommodation really is a repair strategy, triggered e.g. when a definite is used without a retrievable antecedent

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- Similarly for supplements (from Amaral et al., 2007):
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- Similarly for supplements (from Amaral et al., 2007):
 - (21) Joan is crazy. She's hallucinating that some geniuses in Silicon Valley have invented a new brain chip that's been installed in her left temporal lobe and permits her to speak any of a number of languages she's never studied. Joan believes that her chip, which she had installed last month, has a twelve year guarantee.

Taxonomy of Nonconversational Implicatures

A Grice-inspired taxonomy of implicature, leaving out the conversational implicatures (which are nonconventional):



(Here, implicatures with variable commitment status are highlighted.)

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- DyCG is a grammar formalism for modeling language use in context
- It is both *compositional* and *dynamic*: utterances both update the context and depend on it for their own interpretation

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Introducing Dynamic Categorial Grammar (DyCG)

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- It is both *compositional* and *dynamic*: utterances both update the context and depend on it for their own interpretation
- The goal of most dynamic theories is to model anaphora; DyCG aims at implicatures more generally
- It is most similar to the compositional dynamic theories advanced by Beaver (2001) and de Groote (2006)

- Adopting Curry's (1961) distinction between abstract and concrete syntax, DyCG is built upon three logics:
 - Concrete syntax models word order: An instantiation of (simple) type theory with a single nonlogical type s, of strings, that has a concatenation operator with a two-sided identity (the empty string)

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 - Semantics models meaning in context: A dependent type theory with the nonlogical types e (entities), p (propositions), ω (natural numbers)
- It shares the abstract/concrete strategy with de Groote's (2001) Abstract Categorial Grammars and Muskens's (2007) Lambda Grammars

DyCG Grammar Rules

The grammar is a system for deriving *signs*, which, ignoring concrete syntax, are pairs of the form

 $A \; ; b : B \; ,$

where *A* is a formula of abstract syntax, *b* a term of the semantics, and *B* a semantic type.

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where *A* is a formula of abstract syntax, *b* a term of the semantics, and *B* a semantic type. There are only four rules:

$$\vdash A ; b : B \quad (Entry)$$

$$A ; x : B \vdash A ; x : B \quad (Trace)$$

$$\frac{\Gamma, A ; x : B \vdash C ; d : D}{\Gamma \vdash A \multimap C ; (\lambda_x d) : B \to D} \quad (Extract)$$

$$\frac{\Gamma \vdash A \multimap B ; f : C \to D \quad \Delta \vdash A ; c : C}{\Gamma, \Delta \vdash B ; (f c) : D} \quad (Combine)$$

The Underlying Static Semantics I

The underlying static semantics assumes that:

- 1. There is a type w of worlds, and
- 2. For every meaning type *A*, there is some function $@_A : A \to w \to Ext(A)$, that takes each inhabitant of *A* to its extension Ext(A) at a given world

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- But it is *agnostic* about how @p is defined, following Plummer and Pollard (2012)
- In particular, this means there is no need to define an extensional fragment

The Underlying Static Semantics II

The static semantics also axiomatizes these connectives and quantifiers to behave as expected:

> entails : $p \rightarrow p \rightarrow t$ (entailment) true : p (a necessary truth) false : p (a necessary falsehood) $\mathsf{not}: \mathsf{p} \to \mathsf{p}$ (negation) and : $p \rightarrow p \rightarrow p$ (conjunction) implies : $p \rightarrow p \rightarrow p$ (implication) (disjunction) or : $p \rightarrow p \rightarrow p$ forall : $(A \rightarrow p) \rightarrow p$ (universal quantifier) exists : $(A \rightarrow p) \rightarrow p$ (existential quantifier)

Contexts

- A DyCG context is a function from an *n*-ary vector of entities to a proposition.
- The type of *n*-contexts is

$$\mathbf{c}_n =_{\mathrm{def}} \mathbf{e}^n \to \mathbf{p} \; ,$$

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► For example, the following is a 2-context:

 $\vdash \lambda_{x,y}$.(cyclist x) and (bike y) and (ride y x) : c₂

The type c_n shows how dependent types are used in DyCG: an n-context requires n entities to produce a proposition

Contents

- Meanings of declarative utterances are modeled as contents, functions from contexts to pairs of contexts
- ► The type k_n is the type of contents that introduce *n* discourse referents:

 $\mathbf{k}_n =_{\mathrm{def}} \mathbf{c}_m \to (\mathbf{c}_{m+n} \times \mathbf{c}_{m+n})$

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- Inspired by Karttunen and Peters (1979), the first component represents the sense of the expression, and the second its implicature
- For example, It's raining would get the content

 $\vdash \lambda_{c}. \langle \lambda_{\mathbf{x}^{|c|}}. \mathsf{rain}, \lambda_{\mathbf{x}^{|c|}}. \mathsf{true} \rangle : k_{0}$

Dynamic Properties

- Static properties are made dynamic by replacing their entity arguments with vector coordinates
- For example, the unary property cyclist : $e \rightarrow p$ and binary relation ride : $e \rightarrow e \rightarrow p$ are dynamicized as

$$\mathsf{CYCLIST} = \lambda_n \lambda_c. \left\langle \lambda_{\mathbf{x}^{|c|}}.(\mathsf{cyclist}\,\mathbf{x}_n), \lambda_{\mathbf{x}^{|c|}}.\mathsf{true} \right\rangle : \omega \to k_0 \,,$$

and

$$\texttt{RIDE} = \lambda_{mn} \lambda_c \left\langle \lambda_{\mathbf{x}^{|c|}}.(\texttt{ride} \, \mathbf{x}_m \, \mathbf{x}_n), \lambda_{\mathbf{x}^{|c|}}.\texttt{true} \right\rangle : \omega \to \omega \to k_0$$

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 A content modifies the discourse context by being promoted to update, of type

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► The *context change* function CC : k_n → u_n does the promotion by collapsing the sense and implicature content together:

$$cc =_{def} \lambda_{kc} \lambda_{\mathbf{x}^{|c|}, \mathbf{y}^{|k|}} (c \mathbf{x}) and (k c)^{s} \mathbf{x}, \mathbf{y} and (k c)^{i} \mathbf{x}, \mathbf{y},$$

where $(kc)^{s}$ is the sense of *k*, and $(kc)^{i}$ its implicature

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• For example, for some $n : \omega$,

 $\vdash cc(CYCLIST n) = \lambda_c \lambda_{\mathbf{x}^{|c|}} . (c \mathbf{x}) and (cyclist \mathbf{x}_n) and true : \mathbf{u}_0$

Dynamic Connectives I: Conjunction

► The dynamic conjunction of two contents AND : k_m → k_n → k_{m+n} passes to the second conjunct the context updated by the first conjunct:

AND =_{def}
$$\lambda_{hkc} \left\langle \lambda_{\mathbf{x}^{[c]}, \mathbf{y}^{[h]}, \mathbf{z}^{[k]}} \cdot (h c)^{s} \mathbf{x}, \mathbf{y} \text{ and } (k (\operatorname{cc} h c))^{s} \mathbf{x}, \mathbf{y}, \mathbf{z} \right.$$

 $\lambda_{\mathbf{x}^{[c]}, \mathbf{y}^{[h]}, \mathbf{z}^{[k]}} \cdot (h c)^{i} \mathbf{x}, \mathbf{y} \text{ and } (k (\operatorname{cc} h c))^{i} \mathbf{x}, \mathbf{y}, \mathbf{z} \left. \right\rangle$

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- This seems pretty involved, but it just conjoins the two contents in a way that keeps sense and implicature separate
- Example: for natural numbers *m* and *n*,

 $\vdash (\operatorname{CYCLIST} m) \operatorname{AND} (\operatorname{RIDE} n m) \\ \equiv \lambda_c. \langle \lambda_{\mathbf{x}^{|c|}}. (\operatorname{cyclist} \mathbf{x}_m) \text{ and } (\operatorname{ride} \mathbf{x}_n \mathbf{x}_m), \lambda_{\mathbf{x}^{|c|}}. \operatorname{true} \rangle : \mathbf{k}_0$

Dynamic Connectives II: Existential "Quantifier"

► The dynamic existential "quantifier" EXISTS : (ω → k_n) → k_{n+1} doesn't really do any quantifying:

EXISTS =
$$_{def} \lambda_{Dc} . D |c| c^+$$

Here, c^+ is the context just like *c* but with an extra vector coordinate.

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► The dynamic existential "quantifier" EXISTS : (ω → k_n) → k_{n+1} doesn't really do any quantifying:

EXISTS =
$$_{def} \lambda_{Dc} . D |c| c^+$$

Here, c^+ is the context just like c but with an extra vector coordinate.

• Example:

$$\vdash \text{EXISTS CYCLIST} = \lambda_c. \left\langle \lambda_{\mathbf{x}^{|c|}, y}.(\mathsf{cyclist}\, y), \lambda_{\mathbf{x}^{|c|}, y}.\mathsf{true} \right\rangle$$

Dynamic Connectives III: Negation

► Dynamic negation NOT : k_n → k₀ not only negates, it also 'traps' any discourse referents introduced in its scope:

NOT =_{def}
$$\lambda_{kc}$$
. $\langle \lambda_{\mathbf{x}^{|c|}}$.not exists $_{\mathbf{y}^{|k|}} . (kc)^{s} \mathbf{x}, \mathbf{y}, \lambda_{\mathbf{x}^{|c|}} . exists_{\mathbf{y}^{|k|}} . (kc)^{i} \mathbf{x}, \mathbf{y} \rangle$

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Example:

$$\begin{split} & \vdash \operatorname{NOT}\left(\operatorname{EXISTS}\operatorname{CYCLIST}\right) \\ & = \lambda_c \left< \lambda_{\mathbf{x}^{|c|}}. \text{not} \operatorname{exists}_y. (\operatorname{cyclist} y), \lambda_{\mathbf{x}^{|c|}}. \operatorname{exists}_y. \operatorname{true} \right> \\ & \equiv \lambda_c \left< \lambda_{\mathbf{x}^{|c|}}. \text{not} \operatorname{exists} \operatorname{cyclist}, \lambda_{\mathbf{x}^{|c|}}. \operatorname{true} \right> : \mathbf{k}_0 \end{split}$$

Defining a Dynamic Semantics

With conjunction, the existential, and negation defined, the other connectives can be defined in terms of them:

IMPLIES =_{def}
$$\lambda_{hk}$$
.NOT (h AND (NOT k)) : $\mathbf{k}_m \to \mathbf{k}_n \to \mathbf{k}_0$
OR =_{def} λ_{hk} .NOT ((NOT h) AND (NOT k)) : $\mathbf{k}_m \to \mathbf{k}_n \to \mathbf{k}_0$
FORALL =_{def} λ_D .NOT EXISTS n .NOT ($D n$) : ($\omega \to \mathbf{k}_n$) $\to \mathbf{k}_0$

Defining a Dynamic Semantics

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$$\begin{aligned} \text{IMPLIES} =_{\text{def}} \lambda_{hk}.\text{NOT} \left(h \text{ and } (\text{NOT} k) \right) &: \mathbf{k}_m \to \mathbf{k}_n \to \mathbf{k}_0 \\ \text{OR} =_{\text{def}} \lambda_{hk}.\text{NOT} \left((\text{NOT} h) \text{ and } (\text{NOT} k) \right) &: \mathbf{k}_m \to \mathbf{k}_n \to \mathbf{k}_0 \\ \text{FORALL} =_{\text{def}} \lambda_D.\text{NOT} \text{ EXISTS}_n.\text{NOT} \left(D n \right) &: (\omega \to \mathbf{k}_n) \to \mathbf{k}_0 \end{aligned}$$

Then the dynamic generalized determiners are in turn defined in terms of these:

$$A =_{def} \lambda_{DE}.EXISTS_{n}.((D n) AND (E n))$$

EVERY =_{def} \lambda_{DE}.FORALL_{n}.((D n) IMPLIES (E n))
Accounting for some Implicature Data

Outline

Characterizing Sense and Implicature What's the Difference? A Gricean Taxonomy Felicity, Accommodation, and Variabilit A More Fully Fleshed-out Picture

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Anaphora Supplements

Conclusions

Anaphora uses the dynamic generalized determiner

THE =_{def} λ_{DEc} . $\langle (E(\text{the } Dc)c)^{s}, \lambda_{\mathbf{x}^{|c|}}.(((D \text{ THAT } E)(\text{the } Dc)c)^{i}\mathbf{x}) \text{ and } \rangle$ exists! $_{n:\omega_{|c|}}.(c \text{ k-entails } (D n))$

For example, The cyclist rides is modeled by

⊢ THE CYCLIST RIDE $\equiv \lambda_c. \langle \lambda_{\mathbf{x}^{[c]}}.(\mathsf{ride}\,\mathbf{x}_{(\mathsf{the}\,\mathsf{CYCLIST}\,c)}),$ $\lambda_{\mathbf{x}^{|c|}}$.exists! $_{n:\omega_{|c|}}$.(c k-entails (CYCLIST n)) : \mathbf{k}_0

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► For example, *The cyclist rides* is modeled by

 $\vdash \text{THE CYCLIST RIDE} \\ \equiv \lambda_c. \left\langle \lambda_{\mathbf{x}^{|c|}}.(\text{ride } \mathbf{x}_{(\text{the CYCLIST } c)}), \\ \lambda_{\mathbf{x}^{|c|}}.\text{exists!}_{n:\omega_{|c|}}.(c \text{ k-entails } (\text{CYCLIST } n)) \right\rangle : \mathbf{k}_0$

Pronouns and other definites can be defined in terms of THE:

IT $=_{def}$ THE NONHUMAN LANCE $=_{def}$ THE NAMED-LANCE

Donkey Anaphora

Instances of 'donkey anaphora' are captured, for example

(22) Every cyclist that owns a bike_i rides it_i

is modeled as

 $\vdash \text{EVERY}(\text{CYCLIST THAT } \lambda_n.\text{A BIKE}_m.(\text{OWN } m n)) \lambda_n.\text{IT}_m.(\text{RIDE } m n)$ = FORALL_n.(((CYCLIST n) AND EXISTS_m.(BIKE m) AND (OWN m n)) IMPLIES IT_m.(RIDE m n))

This has the sense

 $\lambda_{\mathbf{x}^{|c|}}$.not exists_{y,z}.(cyclist y) and (bike z) and (own z y) and not (ride z y)

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But note that the scope has the implicature

 $\lambda_{\mathbf{x}^{|c|}}.\mathsf{exists}_{m:\omega_{|c|}}.(c \text{ k-entails } (\text{NONHUMAN } m))$

The *merge* function ↑ : (ω → k_n) → (ω → k_n) turns sense content into implicature content

$$\Uparrow =_{\operatorname{def}} \lambda_{Dnc} \cdot \left\langle \lambda_{\mathbf{x}^{|c|}, \mathbf{y}^{|Dn|}} \operatorname{true}, \lambda_{\mathbf{x}^{|c|}, \mathbf{y}^{|Dn|}} \cdot (Dn)^{\operatorname{s}} \mathbf{x}, \mathbf{y} \operatorname{and} (Dn)^{\operatorname{i}} \mathbf{x}, \mathbf{y} \right\rangle$$

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For example, merging a predicativized version of *a cyclist* gives $\vdash \Uparrow(\text{PRED}(\text{A CYCLIST}))$ $\equiv \lambda_{nc}. \langle \lambda_{\mathbf{x}^{[c]}}. \text{true}, \lambda_{\mathbf{x}^{[c]}}. \text{exists}_{y}. (\text{cyclist } y) \text{ and } (y \text{ equals } \mathbf{x}_{n}) \rangle$

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► Then the comma intonation is defined in terms of ↑, as

$$\text{COMMA} =_{\text{def}} \lambda_{DQE}.Q\left((\Uparrow D) \text{ that } E\right)$$

Basic Example with a Nominal Appositive

- As a basic example,
 - (23) Lance, a cyclist, is from Texas.

gets modeled as

- $\vdash (\text{COMMA} (\text{PRED A CYCLIST}) \text{ LANCE}) \text{ FROM-TEXAS}$
- = LANCE ((\Uparrow PRED A CYCLIST) THAT FROM-TEXAS)
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- = LANCE ((\Uparrow PRED A CYCLIST) THAT FROM-TEXAS)
- = The named-lance ((\Uparrow pred a cyclist) that from-texas)
- The sense is that Lance is from Texas:

 $\lambda_{\mathbf{x}^{|c|}}.(\text{from-texas } \mathbf{x}_{(\text{the NAMED-LANCE } c)})$

and the implicature contains the information that he is a cyclist:

 $\lambda_{\mathbf{x}^{|c|}}.exists_y.(cyclist y) \text{ and } (y \text{ equals } \mathbf{x}_{(\text{the NAMED-LANCE } c)})$

Negated Nominal Appositive

- A negated version of (23),
 - (24) It's not true that Lance, a cyclist, is from Texas. is given the semantics

 \vdash NOT ((COMMA (PRED A CYCLIST) LANCE) FROM-TEXAS)

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• Here, the implicature is the same as before:

 $\lambda_{\mathbf{x}^{|c|}}$.exists_y.(cyclist y) and (y equals $\mathbf{x}_{(\text{the NAMED-LANCE }c)})$

but the sense is negated:

 $\lambda_{\mathbf{x}^{|c|}}$.not (from-texas $\mathbf{x}_{(\text{the NAMED-LANCE }c)})$

Supplements

Treatment of Supplements

- This approach can account for non-restrictive relatives, as-parentheticals, 'stacked' appositives, and utterance-final appositives:
 - Lance, who's a cyclist, is from Texas. (25)
 - (26)Lance, as a cyclist, rides every day.
 - (27)Lance, a cyclist, a real go-getter, rides every day.
 - (28)Kim met Lance, a cyclist.
- A similar treatment is given to expressives, like
 - (29)Lance entered the Tour de France, and the damn doper won it.

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- A similar treatment is given to expressives, like
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- It also allows implicature content to interact with the sense content

Sense/Implicature Interaction

- Sometimes sense and implicature interact, as in
 - (30) Lance, a cyclist that has a bike_{*i*}, rides it_{*i*}.

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and so rides it_i is passed a context containing

 $\begin{array}{l} \lambda_{\mathbf{x}^{|c|}}.\mathsf{exists}_{y,z}.(\mathsf{cyclist}\,y) \text{ and } (\mathsf{bike}\,z) \text{ and } (\mathsf{have}\,z\,y) \text{ and } \\ (y \text{ equals } \mathbf{x}_{(\mathsf{the}\,\mathsf{NAMED-LANCE}\,c)}) \end{array}$

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and so rides it_i is passed a context containing

 $\lambda_{\mathbf{x}^{[c]}}$.exists_{*y*,*z*}.(cyclist *y*) and (bike *z*) and (have *z y*) and (*y* equals $\mathbf{x}_{(\text{the NAMED-LANCE } c)}$)

 The anaphora works out, but we have to resort to implementing Roberts's (2003) *weak familiarity*

A Problem: Quantified Supplements

- DyCG does fine with examples like
 - (31) Some cyclist, a doper, won the Tour de France.

A Problem: Quantified Supplements

- DyCG does fine with examples like
 - (31) Some cyclist, a doper, won the Tour de France.
- But for supplements in the scope of quantifiers like
 (32) # No cyclist, a doper, won the Tour de France.
 - It still gives the implicature

 $\lambda_{\mathbf{x}^{|c|}}.\mathsf{exists}_{y,z}.(\mathsf{doper}\,y) \text{ and } (y \text{ equals } z)$

A Solution?

- Nouwen (2007) tries to solve this by requiring quantifiers to introduce plural antecedents, since examples like
 - (33) Every climber, all experienced adventurers, made it to the summit

are fine

- But this approach doesn't seem to extend to
 - (34) No Tibetan Buddhist_i believes that the Dalai Lama, his_i spiritual mentor, would ever bow to Chinese pressure tactics.

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Taking Stock

Some positives:

- A re-examination of the Gricean picture of implicatures leads to a unified account of anaphora and other implicatures
- The formal theory is both compositional and dynamic, extending some ideas originally due to Heim (1982) and Karttunen and Peters (1979)
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Some negatives:

- The problem of quantified appositives seems tough to crack
- Weak familiarity is hard to formalize
- I haven't said anything about how to model speaker vs. non-speaker commitments

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Any ideas?

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Axioms for the Static Semantics I

The *extension type* of a meaning type A is denoted Ext(A).

$$Ext(1) =_{def} 1$$

$$Ext(e) =_{def} e$$

$$Ext(p) =_{def} t$$

$$Ext(A \rightarrow B) =_{def} A \rightarrow Ext(B)$$

$$Ext(A \times B) =_{def} Ext(A) \times Ext(B)$$

The extension functions @ are written infix, similarly to phenogrammatical concatenation, and are subject to the following axioms.

$$\vdash \forall_{w:w}.(* @_1 w) = *$$

$$\vdash \forall_{x:e} \forall_{w:w}.(x @_e w) = x$$

$$\vdash \forall_{f:A \to B} \forall_{w:w}.(f @_{A \to B} w) = \lambda_{x:A}.(f x) @_{A \to B} w$$

$$\vdash \forall_{c:A \times B} \forall_{w:w}.(c @_{A \times B} w) = \langle (\pi_1 c) @_A w, (\pi_2 c) @_B w \rangle$$

Axioms for the Static Semantics II

 $\vdash \forall_{p:p} \forall_{q:p} . (p \text{ entails } q) \Leftrightarrow \forall_{w:w} . ((p @ w) \Rightarrow (q @ w))$ $\vdash \forall_{w \cdot w}$.true @ w $\vdash \forall_{w:w}.\neg(\mathsf{false} @ w)$ $\vdash \forall_{p:p} \forall_{w:w}.((\operatorname{\mathsf{not}} p) @ w) \Leftrightarrow \neg(p @ w)$ $\vdash \forall_{p:p} \forall_{q:p} \forall_{w:w}. ((p \text{ and } q) @ w) \Leftrightarrow ((p @ w) \land (q @ w))$ $\vdash \forall_{p:p} \forall_{q:p} \forall_{w:w}. ((p \text{ implies } q) @ w) \Leftrightarrow ((p @ w) \Rightarrow (q @ w))$ $\vdash \forall_{p:p} \forall_{q:p} \forall_{w:w} . ((p \text{ or } q) @ w) \Leftrightarrow ((p @ w) \lor (q @ w))$ $\vdash \forall_{P:A \to p} \forall_{w:w}. ((\text{forall } P) @ w) \Leftrightarrow \forall_{x:A}. ((Px) @ w)$ $\vdash \forall_{P:A \to p} \forall_{w:w}. ((\mathsf{exists} P) @ w) \Leftrightarrow \exists_{x:A}. ((Px) @ w)$

Contextual Entailment

$$\begin{split} & \vdash \forall_{p:\mathbf{p}} \forall_{q:\mathbf{p}} \forall_{w:\mathbf{w}}.(p \text{ Entails } q) @ w \Leftrightarrow (p \text{ entails } q) \\ & \mathsf{c}\text{-entails} =_{\mathrm{def}} \lambda_{c:\mathbf{c}} \lambda_{d:\mathbf{c}_{\geq |c|}}. \text{forall}_{\mathbf{x}^{|c|}}.(c \mathbf{x}) \text{ Entails exists}_{\mathbf{y}^{|d|-|c|}}.(d \mathbf{x}, \mathbf{y}) \\ & \mathsf{k}\text{-entails} =_{\mathrm{def}} \lambda_{c:\mathbf{c}} \lambda_{k:\mathbf{k}}.c \text{ c-entails} (\operatorname{cc} k c) \end{split}$$

Definitions for Anaphora

the =_{def}
$$\lambda_{D:\omega \to k_m} \lambda_{c:c} \eta_{n:\omega_{|c|}} . c$$
 k-entails $(D n)$
pro =_{def} $\lambda_{D:\omega \to k_m} \lambda_{c:c} \eta_{n:\omega_{|c|}} . c$ k-cons $(D n)$

Additional Dynamic Definitions

THAT =_{def} λ_{DEn} .(Dn) and (En) PRED =_{def} λ_{On} .NOT (NOT ($Q_m.m$ EQUALS n))