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A Higher-Order Theory of Presupposition

Abstract. So-called 'dynamic' semantic theories such as Kamp's discourse representation theory and Heim's file change semantics account for such phenomena as cross-sentential anaphora, donkey anaphora, and the novelty condition on indefinites, but compare unfavorably with Montague semantics in some important respects (clarity and simplicity of mathematical foundations, compositionality, handling of quantification and coordination). Preliminary efforts have been made by Muskens and by de Groote to revise and extend Montague semantics to cover dynamic phenomena. We present a new higher-order theory of discourse semantics which improves on their accounts by incorporating a more articulated notion of *context* inspired by ideas due to David Lewis and to Craige Roberts.

On our account, a context consists of a *common ground* of mutually accepted propositions together with a set of *discourse referents* preordered by relative *salience*. Employing a richer notion of contexts enables us to extend our coverage beyond pronominal anaphora to a wider range of *presuppositional* phenomena, such as the factivity of certain sentential-complement verbs, resolution of anaphora associated with arbitrarily complex definite descriptions, presupposition 'holes' such as negation, and the independence condition on the antecedents of conditionals.

Formally, our theory is expressed within a higher-order logic with natural number type, separation-style subtyping, and dependent coproducts parameterized by the natural numbers. The system of semantic types builds on proposals due to Thomason and to Pollard in which the type of propositions (static meanings of sentential utterances) is taken as basic and worlds are constructed from propositions (rather than the other way around as in standard Montague semantics).

Keywords: discourse, context, anaphora, presupposition, higher-order logic.

1. Aims and Scope

A central notion in linguistic theories of discourse, such as discourse representation theory (DRT; [9, 10]) and file change semantics (FCS; [5, 6, 7]), is that an utterance's interpretation is dynamic: it both depends upon and modifies the discourse context it is situated within. Such theories provide insightful accounts of such phenomena as cross-sentential anaphora and the novelty condition on indefinites (see (A), where *she* can refer to Chiquita but the phrase *a donkey* cannot).

(A) 1. Chiquita$_i$ arrived.

2. $\left\{ \begin{array}{r} \text{She}_i \\ \text{\#A donkey}_i \end{array} \right\}$ brayed.

These theories also capture the notorious 'donkey anaphora' (*Every farmer who owns a donkey*$_i$ *beats it*$_i$; see (Q) in section 2). When they were introduced, accounts of dynamic phenomena such as these were unavailable in the then-predominant natural language semantic research framework established by Montague [16] a decade earlier. On the other hand, as pointed out by Muskens [17, 18], the new dynamic theories compared unfavorably with Montague semantics in some respects. For one thing, such phenomena as quantification and coordination were not handled as elegantly as in Montague semantics. Moreover, the new dynamic frameworks failed to provide an account of the compositionality of meaning analogous to the stepwise semantic interpretation that accompanies a Montagovian syntactic derivation.

As Muskens [18] points out, subsequent efforts to combine the virtues of Montague semantics with those of DRT/FCS all "base[d] themselves on *ad hoc* special purpose logics and ha[d] a more or less baroque underlying mathematics which is not very well understood" (p. 144). In their place, Muskens offered a new formulation of dynamic semantics that made use of no formal resources beyond those of standard higher-order logic (HOL) [1]. However, Muskens' system is not without problems of its own. For one thing, some of the technical details, such as the reconstruction of the DRT notion of accessibility for discourse referents (hereafter, DRs), might be thought by some to preserve the baroque character of the original. A more serious shortcoming is that, although Muskens' approach does handle intersentential and donkey pronominal anaphora, it does not extend to a wider range of presuppositional phenomena (see below) which depend not merely on access to suitable discourse referents but also on some notion of *common ground* (CG, roughly, those propositions which the discourse participants take to be mutually accepted), e.g., factivity of certain sentential-complement verbs, or the independence condition on the antecedents of conditionals. The difficulty, in our view, is that Muskens' notion of context (or, as he calls it, state), is insufficiently rich, consisting essentially of nothing more than an assignment function mapping DRs (or, as he calls them, pigeonholes) to entities.

De Groote [3, 4] offers a different higher-order account of intersentential and donkey anaphora which appears to improve on Muskens' account in certain respects. De Groote dispenses altogether with DRs (or pigeonholes) in favor of what he calls a *left context* consisting of the set of entities available for anaphoric reference. In place of Muskens' higher-order reconstruction of DRT accessibility, de Groote uses *continuations* (or, in his terminology, *right contexts*, which, type-theoretically, are just functions from left contexts to propositions) to make previously introduced entities available to the subsequent discourse. Though technically much simpler than Muskens' approach, de Groote's account is still problematic in certain respects. For one thing, there is no theory of *which* entity a pronoun refers to; instead, the decision is left to an oracular choice function that is supposed to always pick the 'right' antecedent. Alas, there can be no such function, since the choice of antecedent for a pronoun depends not just on what entities have been discussed but also on what has been said about them and in what order. For example, consider the effect of interchanging the first two sentences of the discourse (B).

(B)
1. A donkey$_i$ walked into a bar.
2. A mule$_j$ did too.
3. It$_{\#i/j}$ ordered a Jack Daniels straight up.

Moreover, as with Muskens, there does not seem to be a straightforward way of elaborating de Groote's architecture to account for presuppositions which depend on a notion of CG.

In this paper, we build on our earlier work on dynamics [15] in proposing a new higher-order theory of presupposition, including intersentential and donkey anaphora, which, while incorporating certain elements of both Muskens' and de Groote's theories, also improves on both by employing a richer notion of context. Inspired by the work of Roberts [23, 25], which in turn draws much of its inspiration from Lewis's [14] 'scoreboard' metaphor, our discourse contexts consist of (at least) a preordered set of DRs and a CG. Like Muskens' pigeonholes, our DRs are taken to be abstract semantic objects which do duty in discourse for the actual entities under discussion (whose identity may well be unknown to the discourse participants). The preorder on DRs, called *salience*, models the extent to which DRs are favored to serve as antecedents for subsequent definite anaphora. The CG of a context is a record of the propositions that are taken by the interlocutors to be mutually accepted (at least for the purposes of keeping the discourse going), contributed either linguistically (Roberts' *proffered content*) or nonlinguistically, from sources such as sense data and world knowledge. At this stage, since our aim is to treat a few core presuppositional phenomena, we omit some more complex aspects of Roberts' discourse context such as the questions under discussion, common propositional/question space, moves, and domain goals.

Formally, we express our theory within a classical higher-order logic in the tradition of Church [1], incorporating the axiom of boolean extensionality [8] that identifies biimplication with equality of truth values (type t, corresponding to Church's o). Following Lambek and Scott [13], we employ a natural number type ω as well as separation-style subtyping. Additionally, we make use of dependent coproduct types parameterized by the natural numbers. The basic meaning types are e (entities, corresponding to Church's ι) and p (propositions, in the sense of static meanings of declarative sentence utterances). The idea of taking propositions as basic seems to originate with Thomason [28]; we follow Pollard [20, 21] in axiomatizing the type p as a boolean preorder (and then the type of worlds, if needed, is constructed as the subtype of $\mathrm{p} \rightarrow \mathrm{t}$ consisting of ultrafilters, rather than taking propositions to be sets of worlds as Montague did).

The rest of this paper is organized as follows. Section 2 sketches the range of linguistic phenomena the proposed theory is intended to account for. In section 3, we explain the underlying type system and how it is used to model discourse contexts and dynamic meanings. Section 4 provides illustrative analyses of a range of presuppositional phenomena. And in section 5 we draw some conclusions and promise future work.

2. Some Presuppositional Phenomena

A popular (if not standard) characterization of presupposition, due most notably to Stalnaker [26], is that the presuppositions of a sentence are the conditions on the discourse context that must obtain in order for an utterance of the sentence to be felicitous. The conditions in question in turn are characterized in terms of what the interlocutors can infer (or perhaps, think that they can infer) from the CG. A similar characterization, framed in terms of FCS [5, 6], is that sentence interpretations are partial functions from contexts to contexts, where the context is taken to be the conjunction of the propositions in the CG [27]. To paraphrase Heim's view, what is presupposed by a sentence is a predicate on the set of contexts whose characteristic function is the domain of the function which interprets the sentence. The approach to presupposition that we will endorse is strongly influenced by views such as these, but before trying to articulate it, we consider a few illustrative examples.

Among the simplest cases of presupposition are those standardly discussed under the rubric of *factivity*. Factive verbs such as *suck*, *know* and *regret* presuppose the truth of their complement proposition, while verbs like *think* and *believe* do not, as illustrated by the following single-speaker discourses:

(C) 1. Pedro thinks it's raining.

2. But it's not raining.

(D) 1. It sucks that it's raining.

2. # But it's not raining.

The infelicity of (D) that is not present in (C) arises from the fact that the verb *sucks* presupposes the truth of its complement *it's raining*. This explains the strangeness of the assertion in (D2) that it actually isn't raining. By contrast, the verb *thinks* in (C) imposes no such requirement on its complement.

Presuppositions can sometimes "project out of" certain grammatical constructions, as in the case of negation:

(E) 1. It doesn't suck that it's raining.

2. # But it's not raining.

(F) 1. Pedro says it sucks that it's raining.

2. But it's not (even) raining.

Examples like (E) have motivated the description of negation as a 'hole' for presupposition [12]. Informally speaking, the presupposition that it's raining is "passed" from the scope of the negation in (E1), conflicting with the denial of that presupposition in the following assertion by the same speaker (E2). (On the other hand, that same assertion by a different speaker is felicitous, but amounts to a rejection of the previous utterance by denying its presupposition.) By comparison, the single-speaker discourse in (F) is felicitous because the presupposition of the complement clause of (F1) is not passed up to the root clause (and thence into the subsequent discourse); in Karttunen's [12] terminology, the verb *say* is a 'plug' for presupposition projection.

As (G) illustrates, in a conjunctive sentence, the proffered content of the first conjunct becomes part of the CG for the second conjunct; or, to put it another way, the presuppositions of the second conjunct can be satisfied from the first conjunct. Similarly, as (H) shows, in a conditional sentence, the proffered content of the antecedent becomes part of the CG for the consequent.

(G) 1. It's raining and it sucks that it's raining.

2. # It's not raining and it sucks that it's raining.

(H) 1. If it's raining then it sucks that it's raining.

2. # If it's not raining then it sucks that it's raining.

A further presuppositional property of conditionals, illustrated in (I) and (J), is that the antecedent must be *independent* of the CG, in the sense that neither the proffered content of the antecedent nor its denial should be inferable from the CG.

(I) 1. It's raining.
  2. # If it's raining, then my convertible is getting ruined.

(J) 1. It's not raining.
  2. # If it's raining, then my convertible is getting ruined.

Thus, the infelicity of (I) as a single-speaker discourse arises because the antecedent has already been asserted. (However, (I2) could be uttered by a different speaker to withhold acceptance of the previous utterance, by signaling that its proffered content has not yet been admitted to the CG.) And (J) is infelicitous as a single-speaker discourse for the same reason as (I). However, (J2) could be used to delay acceptance of the previous utterance as its speaker infers the absurd consequences of its denial.

Other familiar instances of presupposition arise in connection with so-called *definite anaphora*. The use of a definite expression such as *the donkey* or the pronoun *it* presupposes that a suitable (anaphoric) antecedent is available (Roberts [24, 25]). The relevant notion of suitability in definiteness presuppositions can be characterized informally in terms of the notions of familiarity and greatest salience. Example (K) uttered out of the blue by a harem dweller illustrates failure of familiarity:

(K) # He thinks it's raining.

Here the anaphora fails for lack of a male DR to resolve to. With definite anaphora, though, familiarity alone is insufficient:

(L) 1. A farmer bought the donkey.
  2. What donkey?
  3. # Just some donkey or other, I don't know which.

Even if the context of utterance for (L) contains multiple donkey-DRs as potent resolution targets for *the donkey*, a particular donkey has to have been made more salient than all the others. In discourse, this salience is accomplished by using an indefinite to introduce a DR that can later be subsequently referenced:

(M) 1. A farmer bought a donkey$_i$ and a mule$_j$.
  2. $\left\{ \begin{array}{r} \text{The donkey}_i \\ \#\text{It}_{i/j} \end{array} \right\}$ brayed.

In (M), the definite anaphora with *the donkey* is successfully resolved to the unique donkey-DR that has been introduced into the discourse by *a donkey*, but the pronominal anaphora with *it* fails because two non-human DRs have been introduced and neither has been made more salient than the other. By contrast, in (N), definite anaphora fails with *the donkey* because the two donkey-DRs which have been introduced are equally salient, but *the gray donkey* is sufficiently specific to resolve to just one of them.

(N) 1. A farmer bought a brown donkey$_i$ and a gray donkey$_j$.

2. $\left\{ \begin{array}{r} \text{The gray donkey}_j \\ \#\text{The donkey}_{i/j} \\ \#\text{It}_{i/j} \end{array} \right\}$ snorted.

There are some issues related to salience that are difficult to describe, and perhaps do not even fall under the purview of linguistic theory but are instead properly considered as true pragmatic effects. Consider the following discourses:

(O) 1. A donkey$_i$ walked in.
2. A mule$_j$ walked in.
3. It$_{\#i/j}$ brayed.

(P) 1. A donkey$_i$ walked in.
2. A mule$_j$ walked in.
3. The donkey$_i$ was sad.
4. It$_{i/\#j}$ brayed.

The use of the definite pronoun *it* in (O) must refer to the mule, whereas the effect of definitizing the donkey in (P3) is that the subsequent *it* must refer to the donkey and not the mule.

Finally, we consider the celebrated phenomenon of *donkey anaphora*. Example (Q) is one of these:

(Q) Every farmer who owns a donkey$_i$ beats it$_i$.

One puzzle to be accounted for is that an indefinite introduced in the restriction of a universal quantificational noun phrase (QNP) can antecede a definite anaphor in the scope of the QNP, as in (Q), but not one in a subsequent sentential utterance (R). A second puzzle about donkey anaphora, traditionally called the strong/weak ambiguity, has to do with a certain indeterminacy about the truth conditions of sentences like (Q): must each farmer beat *every* donkey that s/he owns, or just one of them?

(R) 1. Every farmer who owns a donkey$_i$ beats it$_i$.
2. # It$_i$'s gray.

3. Semantic Modeling in Higher-Order Logic

3.1. Typing and Subtyping

We work in a classical HOL with basic types for truth values (t), entities (e), propositions (p), and natural numbers (ω) with the usual linear order $<$ and successor function **suc**. Besides the usual cartesian-closed type constructors U (unit), $\times$ (product), and $\to$ (exponential), we make use of (separation) subtyping, so that for any type A and formula (boolean term) φ, there is a type $\{x \in A \mid \varphi\}$. In a set-theoretic model, this is interpreted as the subset of the interpretation of A that has $\lambda_x \varphi$ as its characteristic function. An important special case are the natural number subtypes

$$\omega_n =_{\mathrm{def}} \{i \in \omega \mid i < n\}$$

whose members are the first n natural numbers. We write $A \rightharpoonup B$ for the type of partial functions from A to B. (Technically, this is the subtype of the type $(A \times B) \to \mathrm{t}$ consisting of those relations from A to B which are (graphs of) functions.) We also use dependent coproduct types parameterized by ω. For P an ω-parameterized type, we write $\coprod_{n \in \omega} P_n$ for the dependent coproduct whose cofactors are the types P_n.

We observe the following notational conventions: pairing is denoted by $\langle\, ,\, \rangle$ and projections by π and π'. Applications are written $(f\,a)$ rather than $f(a)$. Application and pairing associate to the left, so that e.g. $(f\,a\,b)$ abbreviates $((f\,a)\,b)$, while binding by λ and by quantifiers associates to the right. Parentheses are often eliminated using . in the usual way, e.g. $\lambda_x.M\,N$ abbreviates $\lambda_x(M\;N)$. Outermost parentheses are often omitted altogether. Successive λ-abstractions are often simplified, e.g. $\lambda_{xyz}M$ abbreviates $\lambda_x\lambda_y\lambda_z M$. And finally, in a λ-abstract where the type of the bound variable is a subtype $S = \{x \in T \mid \varphi[x]\}$ of some type T, we often write $\lambda_{x \mid \varphi[x]} M[x/y]$ instead of $\lambda_y M$ with y a variable of type S, even though technically this term is ill-typed.

3.2. Modeling Discourse Contexts

Following Heim, we will model DRs as natural numbers. For each n, we define the type of n-ary **anchors** to be the type of functions from the first

n DRs to entities:

$$\mathrm{a}_n =_{\mathrm{def}} \omega_n \rightarrow \mathrm{e}$$

We introduce a family of constants $\bullet_n : \mathrm{a}_n \rightarrow \mathrm{e} \rightarrow \mathrm{a}_{(\mathbf{suc}\, n)}$ (written infix) axiomatized so as to extend an n-ary anchor to an $(n+1)$-ary one that maps the 'next' DR to a specified entity:

$$\vdash \forall_{n\in\omega}\forall_{a\in\mathrm{a}_n}\forall_{x\in\mathrm{e}}.(a \bullet_n x)\, n = x$$
$$\vdash \forall_{n\in\omega}\forall_{a\in\mathrm{a}_n}\forall_{x\in\mathrm{e}}\forall_{m\in\omega_n}.(a \bullet_n x)\, m = (a\, m)$$

To model relative salience of the first n DRs, we define the type of n-ary **resolutions** to be the subtype of the type of binary relations on ω_n consisting of the preorders:

$$\mathrm{r}_n =_{\mathrm{def}} \{r \in \omega_n \rightarrow \omega_n \rightarrow \mathrm{t} \mid (\mathsf{preo}_n\, r)\}$$

where the constant $\mathsf{preo}_n : (\omega_n \rightarrow \omega_n \rightarrow \mathrm{t}) \rightarrow \mathrm{t}$ is axiomatized so that $(\mathsf{preo}_n\, r)$ says of r that it is a reflexive and transitive relation:

$$\vdash \forall_{n\in\omega}\forall_{r\in\omega_n\rightarrow\omega_n\rightarrow\mathrm{t}}.(\mathsf{preo}_n\, r) = \forall_{i\in\omega_n}.(i\, r\, i) \wedge \forall_{j,k\in\omega_n}.((i\, r\, j) \wedge (j\, r\, k)) \rightarrow (i\, r\, k)$$

The counterpart of $\bullet_n$ for the resolution preorders is $\star_n : \mathrm{r}_n \rightarrow \mathrm{r}_{(\mathbf{suc}\, n)}$. This denotes the function that extends an n-ary resolution to an $(n+1)$-ary one by adding the 'next' DR:

$$\vdash \forall_{n\in\omega}\forall_{r\in\mathrm{r}_n}.n\, (\star_n\, r)\, n$$
$$\vdash \forall_{n\in\omega}\forall_{r\in\mathrm{r}_n}\forall_{m\in\omega_n}.(\neg(m\, (\star_n\, r)\, n)) \wedge \forall_{k\in\omega_n}.(k\, (\star_n\, r)\, m) = (k\, r\, m)$$

Note that the extension of a resolution that results from an application of $\star_n$ adds only one new ordered pair, namely $\langle n, n\rangle$.

To model CGs, we use the type p of propositions and associated constants (propositional connectives) from Pollard's hyperintensional semantics [20], axiomatized so that in a model, the propositions form a boolean preorder under the **entailment** preorder denoted by $\mathsf{entails} : \mathrm{p} \rightarrow \mathrm{p} \rightarrow \mathrm{t}$. The constants and, or, not, $\mathsf{implies}$, and true denote, respectively, a greatest lower bound, least upper bound, complement, relative complement, and greatest element in this preorder. Additionally, we avail ourselves of Pollard's propositional existential quantifier exists; once the type w of worlds has been defined and the notion of the extension of a proposition p at a world w (written $p@w$) introduced, this is axiomatized as follows:

$$\vdash \forall_{P\in\mathrm{e}\rightarrow\mathrm{p}}\forall_{w\in\mathrm{w}}.(\mathsf{exists}\, P)@w = \exists_x.(P\, x)@w$$

We add the propositional relation indep (written infix), which says of two propositions that the first entails neither the second nor the denial of the second:

$$\vdash \forall_{p,q \in \mathrm{p}}.(p \ \mathsf{indep}\ q) = \neg((p \ \mathsf{entails}\ q) \vee (p \ \mathsf{entails}\ (\mathsf{not}\ q))) \quad (1)$$

Dynamic interpretation of utterances depends crucially on a notion of discourse context. We define a type for contexts inspired by (a simplification of) a proposal due to Roberts [23]. For us, a context consists of a set of DRs preordered by salience, together with an anchor of the DRs to entities and a CG made up of (the conjunction of) the propositions about (the anchors of) those DRs which have been accepted so far. Formally, a context is a triple containing (1) an anchor, (2) a resolution preorder for the DRs in the domain of that anchor, and (3) a proposition (the CG). Thus the type of n-ary contexts is defined as follows:

$$\mathrm{c}_n =_{\mathrm{def}} \mathrm{a}_n \times \mathrm{r}_n \times \mathrm{p}$$
$$\mathrm{c} =_{\mathrm{def}} \coprod_{n \in \omega} \mathrm{c}_n$$

We abbreviate the three projections of contexts by $\mathbf{a} : \mathrm{c} \to \mathrm{a}$ (for *anchor*), $\mathbf{r} : \mathrm{c} \to \mathrm{r}$ (for *resolution*) and $\mathbf{p} : \mathrm{c} \to \mathrm{p}$ (for *proposition*) respectively.

For each n-ary context, the natural number n is the 'next' DR. We formalize this notion using the constants $\mathsf{next}_n : \mathrm{c}_n \to \omega$:

$$\vdash \forall_{n \in \omega} \forall_{c \in \mathrm{c}_n}.(\mathsf{next}_n\ c) = n$$

We write $[n]_c$ to abbreviate $(\mathbf{a}\,c\,n)$, the entity to which the DR n is mapped by the anchor of the context c.

$$\vdash \forall_{m \in \omega} \forall_{c \in \mathrm{c}_m} \forall_{n \in \omega_m}.[n]_c = (\mathbf{a}\ c\ n)$$

Thus $[n]_c$ denotes the entity that is the image of the DR n under the anchor of the context c. When no confusion can arise, we usually drop the subscript c, simply writing $[n]$. Finally, we introduce constants $::_n$ and $+$ denoting functions which, respectively, anchor a new DR into a context and conjoin a new proposition into a context's CG:

$$\vdash \forall_{n \in \omega}.\ ::_n = \lambda_{cx}\, \langle (\mathbf{a}\ c) \bullet_n x, \star_n\ (\mathbf{r}\ c), (\mathbf{p}\ c) \rangle$$
$$\vdash + = \lambda_{cp}\, \langle (\mathbf{a}\ c), (\mathbf{r}\ c), (\mathbf{p}\ c)\ \mathsf{and}\ p \rangle$$

Thus the task of updating the context is divided between ::, which handles the DRs being discussed, and +, which manages the contributed content of the discourse. It is important to note that while :: is homographic with a notation in de Groote's dynamic theory [3], it differs in that it extends an anchor by adding a mapping from a new DR to an entity while de Groote's :: just adds an entity to an existing set of entities. Since both :: and + associate to the left, we often omit parentheses around embedded applications involving them.

3.3. Dynamic Propositions and Updates

What proposition is contributed by a sentence depends on the context in which it is uttered. In consideration of this fact, we define the type k (mnemonic for *context-dependent proposition*, hereafter CDP) to be the type of *partial* functions from contexts to propositions.

$$\mathrm{k} =_{\mathrm{def}} \mathrm{c} \rightharpoonup \mathrm{p}$$

Here the partiality arises from the fact that in general sentences have presuppositions and therefore can only be interpreted in contexts which satisfy them. Since our contexts (c) can be thought of as an enrichment of de Groote's left contexts (γ), and our propositions (p) as our counterpart of his type o, our type k in turn corresponds roughly to de Groote's type $\gamma \to o$ of right contexts, and in certain respects plays an analogous role in our theory.

For (dynamic) meanings of declarative sentences, we define the type u (mnemonic for *update*) to be the type of unary operations on CDP's:

$$\mathrm{u} =_{\mathrm{def}} \mathrm{k} \to \mathrm{k}$$

Modulo a reversal of the order of the two arguments, this is our analog of de Groote's type Ω, defined as $\gamma \to (\gamma \to o) \to o$. Updates can also be seen as our closest counterpart to Muskens' [17, 18] *boxes* (a.k.a. dynamic propositions), defined as binary relations between states (where Muskens' states amount to the same thing as our anchors).

To explicate the connection between static and dynamic semantics, we recursively define the types R_n of n-ary **static properties** as follows:

$$\mathrm{R}_0 =_{\mathrm{def}} \mathrm{p}$$
$$\mathrm{R}_{(\mathbf{suc}\, n)} =_{\mathrm{def}} \mathrm{e} \to \mathrm{R}_n$$

In particular, nullary static properties are just propositions.

Next we define the types d_n of n-ary **dynamic properties** in an analogous fashion, but with updates as the base of the recursion in place of (static) propositions, and with DRs as the arguments instead of entities:

$$\mathrm{d}_0 =_{\mathrm{def}} \mathrm{u}$$
$$\mathrm{d}_{(\mathbf{suc}\, n)} =_{\mathrm{def}} \omega \to \mathrm{d}_n$$

In particular, nullary dynamic properties are just updates. Since we most often use the type d_1 (the type of dynamic properties), we usually simplify this type by dropping the subscript, writing d instead.

We now define a family of **dynamicizer** functions $\mathbf{dyn}_n$ from static properties to dynamic ones as follows:

$$\vdash \mathbf{dyn}_0 = \lambda_{pkc}.p \text{ and } (k\ (c + p)) : \mathrm{R}_0 \to \mathrm{d}_0$$
$$\vdash \forall_{n\in\omega}\mathbf{dyn}_{(\mathbf{suc}\, n)} = \lambda_{Rm}.(\mathbf{dyn}_n\ (R\ [m])) : \mathrm{R}_{(\mathbf{suc}\, n)} \to \mathrm{d}_{(\mathbf{suc}\, n)}$$

We use bold, lowercase, sans-serif to notate static propositions (e.g. rain) and smallcaps to notate dynamic propositions (e.g. RAIN). Some examples:

$$\vdash \textsc{rain} = (\mathbf{dyn}_0\ \mathsf{rain}) = \lambda_{kc}.\mathsf{rain}\ \mathsf{and}\ (k\ (c + \mathsf{rain}))$$
$$\vdash \textsc{donkey} = (\mathbf{dyn}_1\ \mathsf{donkey}) = \lambda_{nkc}.(\mathsf{donkey}\ [n])\ \mathsf{and}\ (k\ (c + (\mathsf{donkey}\ [n])))$$
$$\vdash \textsc{own} = (\mathbf{dyn}_2\ \mathsf{own}) = \lambda_{mnkc}.(\mathsf{own}\ [m]\ [n])\ \mathsf{and}\ (k\ (c + (\mathsf{own}\ [m]\ [n])))$$

The dynamicizers **dyn** are designed to interact with dynamic conjunction (see (2), below) so as to ensure that the properties they apply to find their way into the common ground of the subsequent discourse context.

The **staticizer** function **stat** is used to retrieve the CDP corresponding to a dynamic property:

$$\vdash \mathbf{stat} = \lambda_u.u\ \top$$

This function, which is analogous to de Groote's READ [4], takes an update u and passes the trivial CDP $\top =_{\mathrm{def}} \lambda_c\mathsf{true}$ to it. Note that the resulting CDP may be undefined for a given context c if the presuppositions of u are not satisfied in c.

As an example use of the staticizer, we apply **stat** to $\textsc{rain} = (\mathbf{dyn}_0\ \mathsf{rain})$ to retrieve its corresponding CDP:

$$\begin{aligned}(\mathbf{stat}\ \textsc{rain}) &= (\lambda_{kc}.\mathsf{rain}\ \mathsf{and}\ (k\ (c + \mathsf{rain}))\ \top) \\ &= \lambda_c.\mathsf{rain}\ \mathsf{and}\ (\top\ (c + \mathsf{rain})) \\ &= \lambda_c.\mathsf{rain}\ \mathsf{and}\ \mathsf{true} \\ &\equiv \lambda_c.\mathsf{rain}\end{aligned}$$

where $\equiv$ is the relation of intensional equivalence (having the same extension at every world).

Finally, $\downarrow : \mathrm{k} \to \mathrm{c} \to \mathrm{t}$ (written infix) tests whether a context is in the domain of a CDP:

$$\vdash \downarrow = \lambda_{kc}.\mathsf{dom}\; k\; c$$

Here dom denotes the function that maps a CDP to the characteristic function of its domain (as a subset of the set of contexts).

3.4. Core Dynamic Connectives and Quantifier

All of our other dynamic connectives and quantifiers will be based on the core connectives AND, NOT, and quantifier EXISTS. The dynamic conjunction AND : $\mathrm{u} \to \mathrm{u} \to \mathrm{u}$ is used to conjoin utterances into discourses:

$$\vdash \textsc{and} = \lambda_{uvk}\lambda_{c \mid (u\,(v\,k))\downarrow c}.u\;(v\;k)\;c \tag{2}$$

Analogously to the dynamic conjunctions used by Muskens [17] and de Groote [3], this one amounts to composition of updates. Our definition of dynamic conjunction makes the additional requirement that the discourse context be in the domain of the composed updates via the condition $(u\,(v\,k)) \downarrow c$ on the context variable c.

As an example, consider the conjunction of RAIN = $(\mathbf{dyn}_0\;\mathsf{rain})$ with POUR = $(\mathbf{dyn}_0\;\mathsf{pour})$ as might be used in the interpretation of *It rains and it pours*:

$$\begin{aligned}
&\vdash \textsc{rain and pour} : \mathrm{u}\\
=&\lambda_{kc}.\textsc{rain}\;(\textsc{pour}\;k)\;c\\
=&\lambda_{kc}.(\lambda_{kc}(\mathsf{rain\ and}\ k\;(c+\mathsf{rain}))\;\lambda_c(\mathsf{pour\ and}\ (k\;(c+\mathsf{pour}))))\;c\\
=&\lambda_{kc}.\lambda_c(\mathsf{rain\ and}\ (\lambda_c(\mathsf{pour\ and}\ k\;(c+\mathsf{pour}))\;(c+\mathsf{rain})))\;c\\
=&\lambda_{kc}.\lambda_c(\mathsf{rain\ and\ pour\ and}\ k\;(c+\mathsf{rain}+\mathsf{pour}))\;c\\
=&\lambda_{kc}.\mathsf{rain\ and\ pour\ and}\ k\;(c+\mathsf{rain}+\mathsf{pour})
\end{aligned}$$

Note that the common ground of the context passed to the second conjunct POUR contains the static propositional content of the first conjunct (rain). Even though both RAIN and POUR happen to be free of presuppositions in this example, in general this means that presuppositions of the second conjunct can be satisfied by the first.

Our dynamic negation NOT : $\mathrm{u} \to \mathrm{u}$, also analogous to de Groote [3], limits the scope of the propositional negation it introduces to the static content of the update being negated, while passing to the subsequent discourse a common ground updated with the denial of that static content:

$$\vdash \textsc{not} = \lambda_{uk}\lambda_{c \mid (u\,k)\downarrow c}.\mathbf{dyn}_0\;(\mathsf{not}\;(\mathbf{stat}\;u\;c)) \tag{3}$$

The condition $(u\ k) \downarrow c$ on the variable representing the discourse context ensures that the context be able to satisfy the presuppositions of the update that is being dynamically negated. (This point is discussed further in section 4.2, below, on presupposition holes.) We take NOT RAIN, which could be used to interpret *It is not raining*, as an example:

$$\begin{aligned}&\vdash \text{NOT RAIN} : \mathrm{u}\\ =&\lambda_k\lambda_{c\ \mid\ (\text{RAIN}\ k)\downarrow c}.(\mathsf{not}\ (\mathbf{stat}\ \text{RAIN}\ c))\ \mathsf{and}\ (k\ (c + (\mathsf{not}\ (\mathbf{stat}\ \text{RAIN}\ c))))\\ =&\lambda_k\lambda_{c\ \mid\ (\text{RAIN}\ k)\downarrow c}.(\mathsf{not\ rain})\ \mathsf{and}\ (k\ (c + (\mathsf{not\ rain})))\end{aligned}$$

Discourse referents are introduced by the **dynamic existential quantifier** EXISTS : d → u:

$$\vdash \text{EXISTS} = \lambda_{Dkc}.\mathsf{exists}\ \lambda_x.D\ (\mathsf{next}\ c)\ k\ (c :: x) \tag{4}$$

Consider the update EXISTS DONKEY as an example (where DONKEY = $(\mathbf{dyn}_1\ \mathsf{donkey})$):

$$\begin{aligned}&\vdash \text{EXISTS DONKEY} : \mathrm{u}\\ =&\lambda_{kc}.\mathsf{exists}\ \lambda_x.\text{DONKEY}\ (\mathsf{next}\ c)\ k\ (c :: x)\\ =&\lambda_{kc}.\mathsf{exists}\ \lambda_x.(\mathsf{donkey}\ [(\mathsf{next}\ c)]_{c::x})\ \mathsf{and}\ \ (k\ (c :: x + (\mathsf{donkey}\ [(\mathsf{next}\ c)]_{c::x})))\\ =&\lambda_{kc}.\mathsf{exists}\ \lambda_x.(\mathsf{donkey}\ x)\ \mathsf{and}\ \ (k\ (c :: x + (\mathsf{donkey}\ x)))\end{aligned}$$

The effect of EXISTS is to introduce a DR, pass it as an argument to a specified dynamic property, and then make newly introduced DR available in the context passed to the rest of the discourse. Note that we are able to reduce $[(\mathsf{next}\, c)]_{c::x}$ to the entity x because, regardless of the complexity of c itself, x is always the image of $(\mathsf{next}\, c)$ under the anchor of the context $c :: x$.

4. Applications of the Theory

Having laid out the foundations of our dynamic hyperintensional semantic theory, we proceed to use it to treat some of the phenomena discussed in section 2.

4.1. Factivity

To model the presuppositions of factivity demonstrated by examples (C) through (G), we first define the dynamic meaning SUCK : u → u of the factive verb *suck*:

$$\vdash \text{SUCK} = \lambda_{uk}\lambda_{c\ \mid\ (\mathbf{p}\ c)\ \mathsf{entails}\ (\mathbf{stat}\ u\ c)}.\mathsf{suck}\,(\mathbf{stat}\,u\,c)\ \mathsf{and}\ (k\,(c + \mathsf{suck}\,(\mathbf{stat}\,u\,c)))$$

Here suck is the static property (of propositions) of sucking. Note that we could not simply define $\textsc{suck}$ to be the dynamicization of this, even if we enlarged the family of dynamicization functions to cover properties of propositions, because the dynamicization of a static property is always free of presuppositions. The factivity of *suck* is captured by the condition on the variable c that $(\mathbf{p}\ c)$ $\mathsf{entails}$ $(\mathbf{stat}\ u\ c)$. In words, this condition requires that the static propositional content of the complement be entailed by the common ground of the discourse context passed to $\textsc{suck}$. With $\textsc{rain} = (\mathbf{dyn}_0\ \mathsf{rain})$, the dynamic meaning of (D1) is:

$$\vdash \textsc{suck rain} = \lambda_k \lambda_{c \mid (\mathbf{p}\ c)\ \mathsf{entails\ rain}}.(\mathsf{suck\ rain})\ \mathsf{and}\ (k\ (c + (\mathsf{suck\ rain}))) : \mathrm{u} \tag{5}$$

Most importantly, the condition on the variable c in (5) requires the CG of the discourse context to entail that it is raining.

With $\textsc{and}$ as defined in (2), example (G1) is analyzed as:

$$\begin{aligned} &\vdash \textsc{rain and } (\textsc{suck rain}) : \mathrm{u} \\ =&\lambda_{kc}.\textsc{rain}\ ((\textsc{suck rain})\ k)\ c \\ =&\lambda_{kc}.\mathsf{rain\ and}\ (\mathsf{suck\ rain})\ \mathsf{and}\ (k\ (c + \mathsf{rain} + (\mathsf{suck\ rain}))) \end{aligned} \tag{6}$$

Notice that requirement that the CG of the discourse context passed to $(\textsc{suck rain}\ k)$ must entail rain (shown in (5)) is satisfied by the context $c + \mathsf{rain}$ that it is passed by $\textsc{rain}$ in (6). The condition that the context entail rain is also the reason why an interpretation of the discourse in (G2) such as

$$\nvdash (\textsc{not rain})\ \textsc{and}\ (\textsc{suck rain})$$

is ruled out in our analysis.

4.2. Presupposition Holes

As discussed in section 2, negation is sometimes described as a hole for presuppositions because they seem to pass right through it. To demonstrate how our theory accounts for this behavior of negation, we analyze (E) by building on the analysis of the factivity of *suck* in (5).

$$\begin{aligned} &\vdash \textsc{not } (\textsc{suck rain}) : \mathrm{u} \\ =&(\textsc{not}\ \lambda_k \lambda_{c \mid (\mathbf{p}\ c)\ \mathsf{entails\ rain}}.(\mathsf{suck\ rain})\ \mathsf{and}\ (k\ (c + \mathsf{suck\ rain}))) \\ =&\lambda_k \lambda_{c \mid (\textsc{suck rain}\ k)\downarrow c}.(\mathsf{not}\ (\mathsf{suck\ rain}))\ \mathsf{and}\ (k\ (c + \mathsf{not}\ (\mathsf{suck\ rain}))) \end{aligned} \tag{7}$$

This analysis accounts for the infelicity of (E2) because of the requirement that the discourse context be in the domain of $(\textsc{suck rain}\ k)$. As we showed in (5), for any CDP k, the domain of (SUCK RAIN k) includes only those discourse contexts whose CG entails rain. So a continuation of the discourse in (7) with a dynamic meaning of (E2) like NOT RAIN is ruled out since the resulting CG could not have the required entailment.

4.3. The Conditional

Examples (I) and (J) demonstrate that, a conditional utterance is infelicitous if either the antecedent or the antecedent's denial is inferable from the CG (except, in the former case, to signal a rejection). Our dynamic meaning for the dynamic conditional IMPLIES : $\mathrm{u} \to \mathrm{u} \to \mathrm{u}$ (written infix) captures this:*

$$\vdash \text{IMPLIES} = \lambda_{uvk}\lambda_{c|(u\,(v\,k))\downarrow c\ \mathsf{and}\ ((\mathbf{p}\,c)\ \mathsf{indep}\ (\mathbf{stat}\,u\,c))}.\text{NOT}\,(u\ \text{AND}\ (\text{NOT}\,v))\,k\,c$$

The first conjunct of the condition on the discourse context is like the presupposition for AND: it allows presuppositions of the consequent to be satisfied in the antecedent. In the second conjunct of the condition, we invoke indep from (1) to ensure that the static proposition expressed by the antecedent is independent of what is already in the common ground.

With SNOW = $(\mathbf{dyn}_0\,\mathsf{snow})$ and RAIN as in (5) and (6), our analysis of *If it rains, it snows* is as follows:

$$\begin{aligned} &\vdash \text{RAIN IMPLIES SNOW} : \mathrm{u} \\ =&\lambda_k\lambda_{c\,|\,\varphi}.\text{NOT}\,(\text{RAIN AND}\,(\text{NOT SNOW}))\,k\,c \\ =&\lambda_k\lambda_{c\,|\,\varphi}.(\mathsf{not}\,(\mathsf{rain\ and}\,(\mathsf{not\,snow})))\ \mathsf{and}\ (k\,(c + \mathsf{not}\,(\mathsf{rain\ and}\,(\mathsf{not\,snow})))) \end{aligned} \tag{8}$$

where $\varphi = \text{RAIN}\,(\text{SNOW}\,k) \downarrow c\ \mathsf{and}\ ((\mathbf{p}\,c)\ \mathsf{indep\ rain})$ is the condition imposed by IMPLIES on the discourse context. The requirement of the antecedent's independence from the CG is expressed by the statement $(\mathbf{p}\,c)\ \mathsf{indep\ rain}$. In this analysis, the infelicity of (I) and (J) is due to the fact that any discourse context whose CG entails either rain or $(\mathsf{not\,rain})$ is not in the required domain.

4.4. Definite Anaphora

Our work on analyzing definite anaphora in a dynamic setting [15] deals with the presuppositions of familiarity and unique greatest salience exhibited in

*In forthcoming work, we define IMPLIES in a way that gives the 'weak' or 'existential' reading to the conditional, and then define the dynamic universal generalized determiner EVERY (cf. (10), below) in terms of IMPLIES.

examples (K), (L), (M) and (N). In this paper, we expand on our earlier account to include the famous "donkey sentence" in (Q).

Since the dynamic meanings of *farmer*, *donkey*, *owns* and *beats* are available from the dynamicizers $\mathbf{dyn}_1$ and $\mathbf{dyn}_2$, we begin with the dynamic generalized determiners A and EVERY, which are both of type $\mathrm{d} \to \mathrm{d} \to \mathrm{u}$:

$$\vdash \textsc{a} = \lambda_{DE}.\textsc{exists}\ \lambda_n.(D\,n)\ \textsc{and}\ (E\,n) \tag{9}$$

$$\vdash \textsc{every} = \lambda_{DE}.\textsc{not}\ (\textsc{exists}\ \lambda_n.(D\,n)\ \textsc{and}\ (\textsc{not}\ (E\,n))) \tag{10}$$

A key difference between the dynamic generalized determiners is that the indefinite A makes a new DR available to the subsequent discourse, while the universal EVERY does not. This is because the use of the outermost NOT in the definition in (10) constrains the scope of EXISTS, while in (9) there is no such constraint.

To see the dynamic generalized determiners in action, we build on the example given for EXISTS that follows (4), above, to the dynamic meaning of the sentence *A donkey brays*:

$$\begin{aligned}
&\vdash \textsc{a donkey bray} : \mathrm{u}\\
=&\textsc{exists}\ \lambda_n.(\textsc{donkey}\ n)\ \textsc{and}\ (\textsc{bray}\ n)\\
=&\lambda_{kc}.\mathsf{exists}\ \lambda_x.((\textsc{donkey}\ (\mathsf{next}\ c))\ \textsc{and}\ (\textsc{bray}\ (\mathsf{next}\ c)))\ k\ (c :: x)\\
=&\lambda_{kc}.\mathsf{exists}\ \lambda_x.(\mathsf{donkey}\ x)\ \mathsf{and}\ (\mathsf{bray}\ x)\ \mathsf{and}\ (k\ (c :: x + \mathsf{donkey}\ x + \mathsf{bray}\ x))
\end{aligned}$$

(Here, DONKEY = ($\mathbf{dyn}_1$ donkey) and BRAY = ($\mathbf{dyn}_1$ bray).) So the indefinite adds a DR mapped to x into the discourse context and makes the propositions donkey x and bray x available in its common ground, as desired. Contrast this with the dynamic meaning of *Every donkey brays*:

$$\begin{aligned}
&\vdash \textsc{every donkey bray} : \mathrm{u}\\
=&\textsc{not}\ (\textsc{exists}\ \lambda_n.(\textsc{donkey}\ n)\ \textsc{and}\ (\textsc{not}\ (\textsc{bray}\ n)))\\
=&\lambda_k\lambda_{c\,|\,\varphi}.(\mathsf{not}\ (\mathsf{exists}\ \lambda_x.(\mathsf{donkey}\ x)\ \mathsf{and}\ (\mathsf{not}\ (\mathsf{bray}\ x))))\ \mathsf{and}\ (k\ (c + \varpi))
\end{aligned}$$

where $\varphi = (\textsc{every donkey bray}\ k) \downarrow c$ represents the condition on the discourse context introduced by NOT, and

$$\varpi = \mathsf{not}\ (\mathsf{exists}\ \lambda_x.(\mathsf{donkey}\ x)\ \mathsf{and}\ (\mathsf{not}\ (\mathsf{bray}\ x)))$$

represents the proposition added to the context's CG. Unlike the indefinite, the universal does not leave a new DR in the resulting discourse context because NOT discards the context in which it is introduced (see (3), above).

The type e variable x is used and appears in the augmented CG, but its scope is λ-bound in the scope of exists, so it is unavailable to the rest of the discourse.

As in [15], resolving definite anaphora associated with *it* is built into the dynamic meaning of $\textsc{it} : \mathrm{d} \rightarrow \mathrm{u}$:

$$\vdash \textsc{it} = \lambda_{Dkc}.D\,(\mathbf{def}\; c\; \textsc{nonhuman})\; k\; c \tag{11}$$

where $\textsc{nonhuman} = (\mathbf{dyn}_1\,\mathsf{nonhuman})$ and the ω-parameterized definiteness operator $\mathbf{def}_n : \mathrm{c}_n \rightarrow \mathrm{d} \rightarrow \omega_n$ finds the best candidate in the resolution of the discourse context with a given dynamic property:

$$\vdash \mathbf{def}_n = \lambda_{cD}. \bigsqcup_{(\mathbf{r}\,c)} \lambda_{i \in \omega_n}.(\mathbf{p}\,c)\;\mathsf{entails}\;(\mathbf{stat}\;(D\,i)\;c)$$

Here $\bigsqcup$ denotes the unique least upper bound (lub) operation.† When applied to a context c and a dynamic property D, $\mathbf{def}_n$ returns the DR which is the unique greatest DR (relative to the resolution preorder of c) which is entailed by the CG of c to 'have' that dynamic property (provided such a DR exists).

To see how **def** allows $\textsc{it}$ to select the best antecedent from the candidates in the discourse context, we assume that the dynamic meaning of the verb phrase *beats it* from (Q) is $\lambda_j.\textsc{it}\;\lambda_i.\textsc{beat}\;i\;j : \mathrm{d}$. For those readers who are unwilling to take this on faith but who have some familiarity with abstract categorial grammar [2] or lambda grammars [19], we provide the semantic part of a categorial derivation in Figure 1. The gist of this derivation is that the transitive verb is fed hypothetical arguments (traces) for the object and subject, the object hypothesis is withdrawn and the pronoun generalized quantifier 'lowered' into the resulting abstract; and finally the subject hypothesis is withdrawn in preparation for lowering in the subject generalized quantifier *every farmer who owns a donkey.* The reduction of this term in (12) shows that, as desired, the dynamic meaning corresponding to *beats it* is the property of beating the most salient nonhuman DR in the discourse context (as selected by **def**).

$$\begin{aligned} &\vdash \lambda_j.\textsc{it}\;\lambda_i.\textsc{beat}\;i\;j : \mathrm{d} \\ =&\lambda_{jkc}.(\lambda_i(\textsc{beat}\;i\;j)\;(\mathbf{def}\;c\;\textsc{nonhuman}))\;k\;c \\ =&\lambda_{jkc}.(\textsc{beat}\;(\mathbf{def}\;c\;\textsc{nonhuman})\;j)\;k\;c \\ =&\lambda_{jkc}.\mathsf{beat}\;[(\mathbf{def}\;c\;\textsc{nonhuman})]\;[j]\;\mathsf{and}\;k\;(c + \mathsf{beat}\;[(\mathbf{def}\;c\;\textsc{nonhuman})]\;[j]) \end{aligned} \tag{12}$$

†For readability, we suppress a restriction on D to the effect that the set $\lambda_{i\in\omega_n}\;\ldots\;(Di)$ has a unique lub with respect to the preorder $(\mathbf{r}\;c)$.

$$\dfrac{\dfrac{\vdash \text{IT} : \mathrm{d}\to\mathrm{u} \qquad \dfrac{\dfrac{\dfrac{\vdash \text{BEAT}:\mathrm{d}_2 \qquad i:\omega\vdash i:\omega}{i:\omega\vdash(\text{BEAT}\,i):\mathrm{d}} \qquad j:\omega\vdash j:\omega}{i:\omega,j:\omega\vdash(\text{BEAT}\,i\,j):\mathrm{u}}}{j:\omega\vdash\lambda_i.\text{BEAT}\,i\,j:\mathrm{d}}}{j:\omega\vdash(\text{IT}\,\lambda_i.\text{BEAT}\,i\,j):\mathrm{u}}}{\vdash\lambda_j.\text{IT}\,\lambda_i.\text{BEAT}\,i\,j:\mathrm{d}}$$

Figure 1. Proof of $\lambda_j.\text{IT}\,\lambda_i.\text{BEAT}\,i\,j$, where $\text{BEAT} = (\mathbf{dyn}_2\ \mathsf{beat})$ and IT is defined in (11).

Thus the dynamic meaning of *beats it* is capable of selecting a uniquely most salient nonhuman antecedent. We must next ensure that the DR associated with A DONKEY is available in the discourse context used to interpret *beats it.*

The relative pronoun *who* in (Q) must take two properties (in this case, the property of being a farmer and the property of owning a donkey) and conjoin them to make a new property. Our definition of the dynamic meaning $\text{WHO} : \mathrm{d} \to \mathrm{d} \to \mathrm{d}$ reflects this:

$$\vdash \text{WHO} = \lambda_{DEn}.(E\,n)\ \text{AND}\ (D\,n) \tag{13}$$

With the dynamic meaning WHO in place, we have everything we need in order to give an analysis of (Q) that accounts for the behavior of the indefinite *a donkey* and the definite anaphora associated with *it.* The dynamic meaning of (Q) is the following update:

$$\vdash (\text{EVERY}\,(\text{WHO}\,\lambda_j(\text{A DONKEY}\,\lambda_i(\text{OWN}\,i\,j))\,\text{FARMER})\,\lambda_j.\text{IT}\,\lambda_i.\text{BEAT}\,i\,j) \tag{14}$$

A proof of this term is given in Figure 2. We proceed step by step through the reduction of (14) in order to show how a suitable antecedent for the definite pronoun *it* is introduced into the discourse context passed to the dynamic meaning of BEATS IT. We begin with WHO, which (as (13) and Figure 2 show) gives rise to the following term:

$$\begin{aligned} &\vdash (\text{WHO}\,\lambda_j(\text{A DONKEY}\,\lambda_i(\text{OWN}\,i\,j))\,\text{FARMER}) : \mathrm{d} \\ =&\lambda_n.(\text{FARMER}\,n)\ \text{AND}\ (\text{A DONKEY}\,\lambda_i.\text{OWN}\,i\,n) \\ =&\lambda_n.(\text{FARMER}\,n)\ \text{AND}\ (\text{EXISTS}\,\lambda_m.(\text{DONKEY}\,m)\ \text{AND}\ (\text{OWN}\,m\,n)) \\ =&\lambda_{nkc}.(\mathsf{farmer}\,[n])\ \mathsf{and}\ (\mathsf{exists}\,\lambda_x.(\mathsf{donkey}\,x)\ \mathsf{and}\ (\mathsf{own}\,x\,[n])\ \mathsf{and}\ (k\,\kappa)) \end{aligned} \tag{15}$$

where $\kappa = c + (\mathsf{farmer}\,[n]) :: x + (\mathsf{donkey}\,x)$ represents the modified context that results from (15). Note that the new discourse context contains an

$$
\dfrac{\vdash \textsc{who} : \mathrm{d} \to \mathrm{d} \to \mathrm{d} \qquad \dfrac{\dfrac{\dfrac{\vdash \textsc{a} : \mathrm{d} \to \mathrm{d} \to \mathrm{u} \quad \vdash \textsc{donkey} : \mathrm{d}}{\vdash (\textsc{a donkey}) : \mathrm{d} \to \mathrm{u}} \qquad \dfrac{\dfrac{\dfrac{\vdash \textsc{own} : \mathrm{d}_2 \quad i : \omega \vdash i : \omega}{i : \omega \vdash (\textsc{own}\, i) : \mathrm{d}} \quad j : \omega \vdash j : \omega}{i : \omega, j : \omega \vdash (\textsc{own}\, i\, j) : \mathrm{u}}}{j : \omega \vdash \lambda_i.\textsc{own}\, i\, j : \mathrm{d}}}{j : \omega \vdash (\textsc{a donkey}\ \lambda_i.\textsc{own}\, i\, j) : \mathrm{u}}}{\vdash \lambda_j.\textsc{a donkey}\ \lambda_i.\textsc{own}\, i\, j : \mathrm{d}}}{\dfrac{\vdash (\textsc{who}\ \lambda_j.\textsc{a donkey}\ \lambda_i.\textsc{own}\, i\, j) : \mathrm{d} \to \mathrm{d} \qquad \vdash \textsc{farmer} : \mathrm{d}}{\vdash (\textsc{who}\ \lambda_j(\textsc{a donkey}\ \lambda_i(\textsc{own}\, i\, j))\ \textsc{farmer}) : \mathrm{d}}}
$$

$$
\dfrac{\dfrac{\vdash \textsc{every} : \mathrm{d} \to \mathrm{d} \to \mathrm{u} \qquad \begin{array}{c}\vdots \\ \vdash (\textsc{who}\ \lambda_j(\textsc{a donkey}\ \lambda_i(\textsc{own}\, i\, j))\ \textsc{farmer}) : \mathrm{d}\end{array}}{\vdash (\textsc{every}\ (\textsc{who}\ \lambda_j(\textsc{a donkey}\ \lambda_i(\textsc{own}\, i\, j))\ \textsc{farmer})) : \mathrm{d} \to \mathrm{u}} \qquad \begin{array}{c}\vdots \\ \vdash \lambda_j.\textsc{it}\ \lambda_i.\textsc{beat}\, i\, j : \mathrm{d}\end{array}}{\vdash (\textsc{every}\ (\textsc{who}\ \lambda_j(\textsc{a donkey}\ \lambda_i(\textsc{own}\, i\, j))\ \textsc{farmer})\ \lambda_j.\textsc{it}\ \lambda_i.\textsc{beat}\, i\, j) : \mathrm{u}}
$$

Figure 2. Proof tree for the term in (14). Here, FARMER $= (\mathbf{dyn}_1\ \mathsf{farmer})$, OWN $= (\mathbf{dyn}_2\ \mathsf{own})$, DONKEY $= (\mathbf{dyn}_1\ \mathsf{donkey})$, and EVERY, WHO, and A are as defined in (10), (13), and (9), respectively. The subproof of $\lambda_j.\textsc{it}\ \lambda_i.\textsc{beat}\, i\, j : \mathrm{d}$ is given in Figure 1.

extra DR that is mapped to the variable $x : \mathrm{e}$. In the next step, the term in (15) is supplied as the first argument to EVERY:

$$
\begin{aligned}
&\vdash (\textsc{every}\ (\textsc{who}\ \lambda_j(\textsc{a donkey}\ \lambda_i(\textsc{own}\, i\, j))\ \textsc{farmer})) : \mathrm{d} \to \mathrm{u} \\
&= \lambda_E.\textsc{not}\ (\textsc{exists}\ \lambda_n.((\textsc{farmer}\, n)\ \textsc{and}\ (\textsc{a donkey}\ \lambda_i(\textsc{own}\, i\, n))) \\
&\qquad \textsc{and}\ (\textsc{not}\ (E\, n))) \\
&= \lambda_E.\textsc{not}\ (\textsc{exists}\ \lambda_n.((\textsc{farmer}\, n) \\
&\qquad \textsc{and}\ (\textsc{exists}\ \lambda_m.(\textsc{donkey}\, m)\ \textsc{and}\ (\textsc{own}\, m\, n)))\ \textsc{and}\ (\textsc{not}\ (E\, n)))
\end{aligned} \tag{16}
$$

Note that the discourse referent introduced by A DONKEY is available in the discourse context passed to $(\textsc{not}\ (E\ n))$ because of the definition of AND in (2). The dynamic property $\lambda_j.\textsc{it}\ \lambda_i.\textsc{beat}\ i\ j$ is then passed as the next argument to EVERY, producing the final result (14), which reduces as follows:

$$
\begin{aligned}
&\vdash (\textsc{every}\ (\textsc{who}\ \lambda_j(\textsc{a donkey}\ \lambda_i(\textsc{own}\, i\, j))\ \textsc{farmer})\ \lambda_j.\textsc{it}\ \lambda_i.\textsc{beat}\, i\, j) : \mathrm{u} \\
&= \textsc{not}\ (\textsc{exists}\ \lambda_n.((\textsc{farmer}\, n) \\
&\qquad \textsc{and}\ (\textsc{exists}\ \lambda_m.(\textsc{donkey}\, m)\ \textsc{and}\ (\textsc{own}\, m\, n))) \\
&\qquad \textsc{and}\ (\textsc{not}\ (\textsc{it}\ \lambda_i.\textsc{beat}\, i\, n))) \\
&= \lambda_k \lambda_{c\,|\,\varphi}.(\mathsf{not}\ (\mathsf{exists}\ \lambda_x((((\mathsf{farmer}\ x) \\
&\qquad \mathsf{and}\ (\mathsf{exists}\ \lambda_y.(\mathsf{donkey}\ y)\ \mathsf{and}\ (\mathsf{own}\ y\ x))) \\
&\qquad \mathsf{and}\ (\mathsf{not}\ (\mathsf{beat}\ [(\mathbf{def}\ \kappa\ \textsc{nonhuman})]\ x)))))\ \mathsf{and}\ k\ (c + \varpi)
\end{aligned}
$$

Here again, φ is the condition imposed by NOT on the discourse context:

$$\begin{aligned}\varphi = ((&\text{EXISTS } \lambda_n((\text{FARMER } n) \\ &\text{AND } (\text{EXISTS } \lambda_m((\text{DONKEY } m) \text{ AND } (\text{OWN } m\, n))) \\ &\text{AND } (\text{NOT } (\text{IT } \lambda_i.\text{BEAT } i\, n)))) \; k) \downarrow c\end{aligned}$$

The symbol κ here represents the modifications to the CG that result from the first argument to EVERY:

$$\kappa = c :: x + (\mathsf{farmer}\ x) :: y + (\mathsf{donkey}\ y) + (\mathsf{own}\ y\ x)$$

Since κ contains the information that y is a donkey, the definiteness operator **def** is able to pick the DR that is the preimage of y out from the context κ it is passed as the most salient nonhuman entity (as detailed in the expansion of the dynamic meaning of *beats it* in (12), above). This means we can reduce (14) a step further to obtain the final term:

$$\begin{aligned}\lambda_k \lambda_{c \mid \varphi}.(&\mathsf{not}\ (\mathsf{exists}\ \lambda_x((\mathsf{farmer}\ x)\ \mathsf{and}\ (\mathsf{exists}\ \lambda_y((\mathsf{donkey}\ y)\ \mathsf{and}\ (\mathsf{own}\ y\ x) \\ &\mathsf{and}\ (\mathsf{not}\ (\mathsf{beat}\ y\ x))))))))\ \mathsf{and}\ k\ (c + \varpi)\end{aligned}$$

Finally, ϖ represents the modifications to the CG made by the entire sentence:

$$\begin{aligned}\varpi = \mathsf{not}\ (&\mathsf{exists}\ \lambda_x.(\mathsf{farmer}\ x)\ \mathsf{and}\ (\mathsf{exists}\ \lambda_y.(\mathsf{donkey}\ y)\ \mathsf{and}\ (\mathsf{own}\ y\ x) \\ &\mathsf{and}\ (\mathsf{not}\ (\mathsf{beat}\ y\ x))))\end{aligned}$$

The reduction of this analysis of (Q) demonstrates how our theory provides a means for a DR introduced in an embedded clause (here *a donkey*) to be referenced later in a discourse, and a means for resolving the correct DR from the definite pronoun *it.*

Another appealing aspect of the analysis presented here is that while it allows a DR introduced by an indefinite to be referenced anaphorically from outside its local clause, this access is appropriately constrained. Consider the infelicity in (R), where a new attempt is made to reference the DR introduced by *a donkey.* Our theory captures the infelicity of such discourses by making the newly introduced DR unavailable outside the scope of *every.* This is because the definition of NOT in (3) ensures that the context passed by (14) to the following discourse is $c + \varpi$, which does not contain either of the DRs introduced inside (14), only the ones already present in the context c. The resulting CG contains the proposition contributed by (14), which uses the variables x and y, but both of these variables are λ-bound within the scope of an exists.

5. Conclusion

Apart from describing the introduction and subsequent accessibility of discourse referents, a central concern of dynamic theories of discourse is how to handle the conditions placed on the discourse context by utterances, i.e., their presuppositions. In this paper, we presented a dynamic theory rooted in well-understood formal and semantic foundations that, as we demonstrated, is capable of handling not only discourse referents but also a wide range of presuppositional phenomena. This theory combines desirable attributes from both static Montague-style semantics and dynamic semantics in the tradition of DRT and FCS in that it accounts for dynamic phenomena such as inter-sentential anaphora in a compositional manner. The result is that interpretations of discourses are built up from utterance interpretations in a way that takes both the asserted and presuppositional aspects of their meanings into account. The advantage of our theory over previous attempts to integrate Montague semantics with DRT/FCS stems from its enriched notion of context, which is expanded from the usual set of DRs to include both a salience preorder on the set of DRs and a common ground of the combined mutually accepted content of the preceding utterances. The failure of utterances whose presuppositions are not satisfied to express propositions is captured in our theory by the choice to model sentence meanings as partial functions on contexts.

There is no shortage of issues that remain to be addressed in future work (some in collaboration with Elizabeth Smith). These can be divided into three categories: formal issues, framework issues, and empirical issues.

On the purely formal side, the underlying logic needs be more precisely specified. We are working in a higher-order logic 'along the lines of' Lambek and Scott's intuitionistic type theory, but some of the adaptations we have made need to be more fully spelled out. Some of these are exercises for Lambek and Scott [13], such as the strengthening of their type theory to allow for all exponentials A^B, not just those where $B = \mathrm{t}$ (p. 133, exercise 4), and the topos generalization of partial functions (p. 152, exercise 1). Additionally, we need to establish that the incorporation of dependent coproducts indexed by the natural number type works as advertised. And we have to show that the specific calculations our analyses depend upon are not adversely affected by decidability issues arising from the use of separation-style subtyping.

In connection with the framework, our modelling of contexts needs further development. Among the more pressing concerns are the following:

1. Common grounds must contain not merely a record of what has been

said and accepted, but also shared encyclopedic background knowledge and what is publicly perceptible.

2. We have said nothing thus far about how the resolution preorder gets updated, e.g. that resuming a discourse referent by a definite description renders it an easier target for subsequent pronominal anaphora.
3. Our handling of presupposition in terms of entailments of the common ground is a popular but unrealistic simplification. Instead, we need a notion of **practical entailment** (and related notions of **practical consistency** and **practical independence**) distinct from true semantic entailment that models the kinds of inferences that interlocutors can be expected to draw in real time.
4. We must account for the fact that what is proffered by utterances is subject to rejection (e.g. because acceptance would render the common ground (practically) inconsistent); but that even rejected utterances can provide antecedents for subsequent definite anaphora.

Finally, we need empirical coverage which is at once broader and more finely grained. For example, we are developing an account of weak/strong ambiguities wherein only the weak readings arise from the lexical semantics of determiners, with (so-called) strong readings being practically entailed by the weak ones in conjunction with background assumptions of 'consistency' (in the specialized sense that, e.g. farmers treat their donkeys in a consistent way [11]). We have to explain why it is that some species of (what have been called) presupposition triggers (PTs), such as pronominal anaphora and *too*, resist accommodation, while other, so-called 'informative' PTs (such as possessives and clefts) are more forgiving. Additionally, we must account for the relative ease with which the (putative) presuppositions of certain classes of (putative) PTs are cancelled (or perhaps more appropriately, simply do not arise); among these are verbs with preparatory phase (e.g. *win*), aspectual verbs (such as *begin*, *continue*, and *quit*), and certain factives (such as *know* and *regret*). And finally, we wish to provide an account of implications (including those classified as conventional implicatures by Potts [22], such as expressives, appositives, and nonrestrictive relatives), which, roughly speaking, arise from not-at-issue speaker commitments and obligatorily project to the top-level context.

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References

[1] Church, Alonzo, 'A formulation of the simple theory of types', *Journal of Symbolic Logic*, 5 (1940), 56–68.

[2] de Groote, Philippe, 'Towards abstract categorial grammars', in *Association for Computational Linguistics, 39th Annual Meeting and 10th Conference of the European Chapter, Proceedings of the Conference*, 2001.

[3] de Groote, Philippe, 'Towards a Montagovian account of dynamics', in *Proceedings of Semantics and Linguistic Theory 16*, 2006.

[4] de Groote, Philippe, 'Typing binding and anaphora: Dynamic contexts as $\lambda\mu$-terms', Presented at the ESSLLI Workshop on Symmetric Calculi and Ludics for Semantic Interpretation, 2008.

[5] Heim, Irene, *The Semantics of Definite and Indefinite Noun Phrases*, Ph.D. thesis, University of Massachusetts, Amherst, 1982.

[6] Heim, Irene, 'File change semantics and the familiarity theory of definiteness', in *Meaning, Use and the Interpretation of Language*, Walter de Gruyter, Berlin, 1983.

[7] Heim, Irene, 'On the projection problem for presuppositions', in M. Barlow, D. Flickinger, and M. Westcoat, (eds.), *WCCFL2: Second Annual West Coast Conference on Formal Linguistics*, Stanford University, Stanford, California, 1983.

[8] Henkin, Leon, 'Completeness in the theory of types', *Journal of Symbolic Logic*, 15 (1950), 81–91.

[9] Kamp, Hans, 'A theory of truth and semantic representation', in J. Groenendijk, T. Janssen, and M. Stokhof, (eds.), *Formal Methods in the Study of Language*, Mathematisch Centrum, Amsterdam, 1981.

[10] Kamp, Hans, and Uwe Reyle, *From Discourse to Logic*, Kluwer Academic Publishers, Dordrecht, 1993.

[11] Kanazawa, Makoto, 'Weak vs. strong readings of donkey sentences and monotonicity inference in a dynamic setting', *Linguistics and Philosophy*, 17 (1994), 2, 109–158.

[12] Karttunen, Lauri, 'Presuppositions of compound sentences', *Linguistic Inquiry*, 4 (1973), 2, 169–193.

[13] Lambek, Joachim, and Phil Scott, *Introduction to Higher-Order Categorical Logic*, Cambridge University Press, Cambridge, 1986.

[14] Lewis, David, 'Scorekeeping in a language game', in R. Baüerle, U. Egli, and A. von Stechow, (eds.), *Semantics from a Different Point of View*, Springer, Berlin, 1979.

[15] Martin, Scott, and Carl Pollard, 'Hyperintensional dynamic semantics: Analyzing definiteness with enriched contexts', in *Proceedings of the 15th Conference on Formal Grammar*, in press. To appear in *Springer Lecture Notes in Computer Science*.

[16] Montague, Richard, 'The proper treatment of quantification in ordinary English', in K. Hintikka, J. Moravcsik, and P. Suppes, (eds.), *Approaches to Natural Language*,

D. Reidel, Dordrecht, 1973.

[17] Muskens, Reinhard, 'Categorial grammar and discourse representation theory', in *Proceedings of COLING*, 1994.

[18] Muskens, Reinhard, 'Combining Montague semantics and discourse representation', *Linguistics and Philosophy*, 19 (1996), 143–186.

[19] Muskens, Reinhard, 'Separating syntax and combinatorics in categorial grammar', *Research on Language and Computation*, 5 (2007), 267–285.

[20] Pollard, Carl, 'Hyperintensional questions', in *Proceedings of the 15th Workshop on Logic, Language, Information, and Computation (WoLLIC '08)*, no. 5110 in Springer Lecture Notes in Artificial Intelligence, 2008, pp. 261–274.

[21] Pollard, Carl, 'Hyperintensions', *Journal of Logic and Computation*, 18 (2008), 2, 257–282.

[22] Potts, Christopher, *The Logic of Conventional Implicatures*, Oxford University Press, 2005.

[23] Roberts, Craige, 'Information structure in discourse: Towards an integrated formal theory of pragmatics', in *Papers in Semantics*, no. 49 in Working Papers in Linguistics, Ohio State University Department of Linguistics, 1996.

[24] Roberts, Craige, 'Uniqueness in definite noun phrases', *Linguistics and Philosophy*, 26 (2003), 3, 287–350.

[25] Roberts, Craige, 'Pronouns as definites', in M. Reimer, and A. Bezuidenhout, (eds.), *Descriptions and Beyond*, Oxford University Press, 2004.

[26] Stalnaker, Robert, 'Presuppositions', *Journal of Philosophical Logic*, 2 (1973), 4, 447–457.

[27] Stalnaker, Robert, 'Assertion', *Pragmatics*, Syntax and Semantics 9 (1978), 315–332.

[28] Thomason, Richmond, 'A model theory for propositional attitudes', *Linguistics and Philosophy*, 4 (1980), 47–70.

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